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Some Problems on Time Change of Gravity

Part 5. On Free Oscillations of the Earth Observed at the Time of the Chilean Earthquake on May 22, 1960

By

Ichiro NAKAGAWA

Abstract

A theoretical investigation on free oscillations of the earth has been made for a long time. On the contrary, an observation of the free oscillations of the earth was carried out by H. Benioff alone at the time of the Kamchatka earthquake on November 4, 1952. But the free oscillations of the earth excited by the great Chilean earthquake on May 22, 1960, were mainly observed in America, and their existence was confirmed to such an extent as it admitted of no doubt.

In the present article, a trial to detect free oscillations of the earth from records obtained working with Askania gravimeter No. 111 at that time, for the purpose of observation of the tidal variation of gravity at Kyoto, is in detail described. Readings of the records are made at 2-minute intervals. 1480 read values thus obtained are filtrated by a high-pass filter and analysed by Fourier's method. Free spheroidal oscillations of the earth with azimuthal wave numbers $n=2$, 3, 4, 0 and 5, seem to be detectable by the present spectral analysis. The obtained periods corresponding to these earth's oscillations, are in good agreement with those observed in America and, furthermore, with periods predicted by the theoretical investigations. The gravity variation with a period of 53.4 minutes is about 0.58 microgals in amplitude and attains its negative maximum at the origin time of the earthquake. The corresponding displacement amplitude is estimated to be about 0.52 centimetres.

1. Introduction

The history of a theoretical investigation on free oscillations of the earth could be traced back to the latter part of the nineteenth century. An early investigation on this problem was discussed for a homogeneous earth by P. Jaerisch (**1**), H. Lamb (**2**), T. J. Bromwich (**3**), J. H. Jeans (**4**), A. E. H. Love (**5**) and other theoreticians.

On the other hand, the first commemorable observation of free oscillations of the earth was carried out by H. Benioff (**6, 7**). He found out a wave with 57-minute period recorded in fluctuation of the zero line on his strain records at the time of the Kamchatka earthquake on November 4, 1952 and regarded it as a representation of the free oscillations of the earth. At the time of a deep focus earthquake occured on November 19, 1954 in the northwestern part of the Japan Sea, long period oscillations in a range of 40 to 70 minutes were observed simultaneously with tiltmeters at two stations, about 90 kilometres apart, by E. Nishimura and others (**8, 9**).

With impetus of the Benioff's report and the latest remarkable progress of an electronic computing machine, a more thorough theoretical investigation on free oscillations of the earth has been made by many researchers (**10–23**) since 1958, and at present such an investigation is being made even for the moon (**24, 25, 26**).

Apace from the time when H. Benioff observed the Kamchatka earthquake, the accuracy of observation was largely increased recently by a remarkable progress of instrument. Especially, instruments became of better equipment since a good opportunity afforded by instance of the "International Geophysical Year" and continuous observations had been made with such instruments at many stations in the world. A great Chilean earthquake then occured. Needless to say that this earthquake was observed with many instruments at work and an existence of the free oscillations of the earth, was confirmed to such an extent as it admitted of no doubt.

Observations of free oscillations of the earth excited by the Chilean earthquake on May 22, 1960, were mainly been reported in America. Observation stations and instruments in them the free oscillations of the earth were observed, were shown in Table 5.1 with epicentral data of the earthquake. The power spectra obtained by these observations showed a splendid agreement one another and their peaks were in marvellously good agreement with

Table 5.1. Epicentral data and observation stations

Epicentre $38° S, 73\frac{1}{2}° W$

Origin time 19h 11m 20s, May 22, 1960 (UT)

Magnitude $8\frac{1}{4} - 8\frac{1}{2}$

Observation station	Instrument	Author
Isabella (California)	Benioff strain seismograph	H. Benioff, F. Press & S. Smith (27)
Ñaña (Peru)	Benioff strain seismograph	
Ogdensburg (New Jersey)	Lamont strain seismograph	L. E. Alsop, G. H. Sutton & M. Ewing (28)
Palisades (New York)	Pendulum seismograph	
Chester (New Jersey)	Columbia vertical seismograph	B. P. Bogert (29)
Los Angeles (California)	LaCoste-Romberg tidal gravimeter	N. F. Ness, J. C. Harrison & L. B. Slichter (30)
Tiefenort (East Germany)	Pendulum seismograph	W. Buchheim & S. W. Smith (31)

periods predicted by the theoretical investigations.

2. Observations

At the time of the Chilean earthquake of May 22, 1960, two sets of Askania gravimeters were working for the purpose of observing the tidal variation of gravity at Kyoto (**32**). One was an Askania Gs-11 gravimeter No. 111 belonging to the Disaster Prevention Research Institute of Kyoto University and was working since July 1959. The other was an Askania Gs-11 gravimeter No. 105 belonging to the Geographical Survey Institute. The latter gravimeter was installed side by side in the same room as the former, and a simultaneous observation of the tidal variation of gravity had been made by using these two gravimeters since January 1960.

The observation room was an underground special room of the Geophysical Institute of Kyoto University where was the International Reference Station of Gravity in Japan, and the temperature and humidity in this room were maintained by means of a thermostatically controlling apparatus throughout a year as 19.5°C±0.2°C and 55%±1% respectively. Both gravimeters were installed at a distance of two metres. The position of the observation room was shown in Table 5.2.

Table 5.2. Description of the observation station

Observation station :	Kyoto
Latitude	35° 01.8′ N
Longitude	135° 47.2′ E
Height	59.9 metres above mean sea level
Depth	2.4 metres below ground surface
Observation room	International Reference Station of Gravity in Japan (Geophysical Institute of Kyoto University)

Records of gravity change at the time of the Chilean earthquake obtained by both gravimeters were shown in Fig. 5.1. The width of recording paper was 20 centimetres and its speed was 8 millimetres per hour. Time-marks were recorded at every hour on the recording paper and their accuracy was always held within 30 seconds. The ratio of sensitivities of both gravimeters was about 4 : 3, but there was a considerable difference in their damping constants. Nevertheless, as could easily be seen from Fig. 5.1, a movement

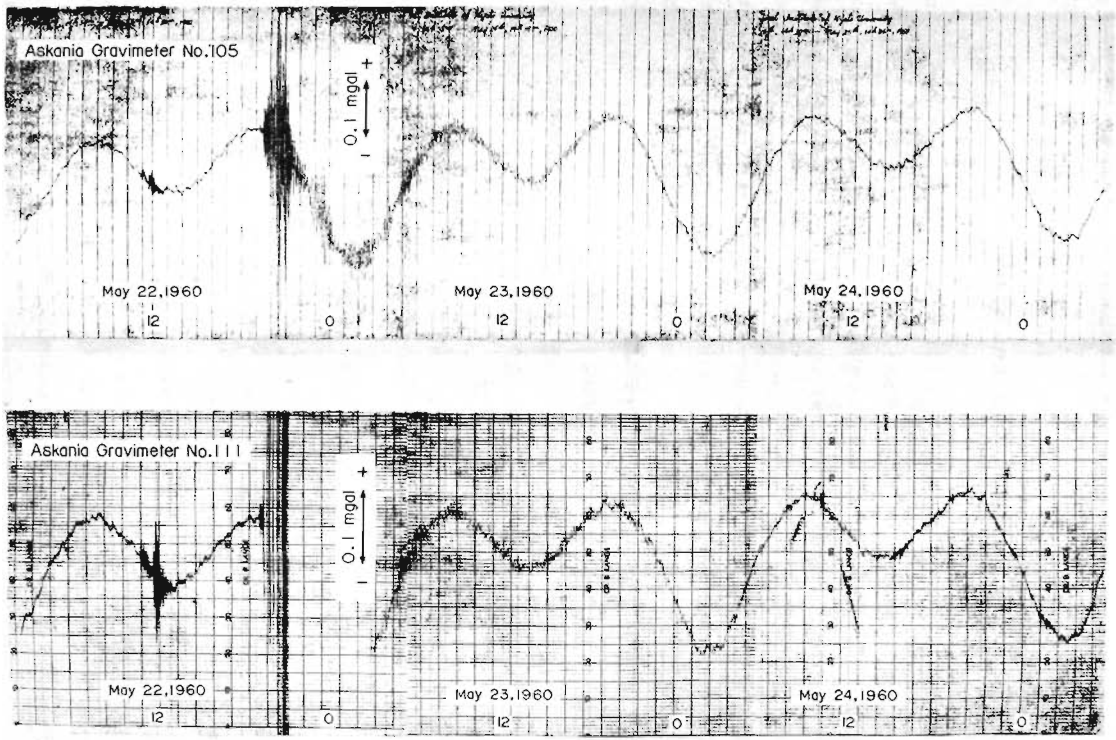

Fig. 5.1. Records of gravity change at the time of the Chilean earthquake of May 22, 1960 simultaneously observed with two Askania gravimeters at Kyoto.
(Size about 1/4 of the original records. Hour-marks in UT.)

of the recorded curves obtained by both gravimeters was perfectly similar over a period of a few days after occurrence of the earthquake. Oscillations so large as to scale out from the recording paper were recorded by the Askania gravimeter No. 111 over a period of a few hours from the arrival time of the seismic wave.

3. Method of analysis

Records obtained with the Askania gravimeter No. 111 were used for the present analysis to detect free oscillations of the earth. During several hours of the arrival time, the recorded curve scaled out from the recording paper, and it was therefore impossible to make readings. Except those durations, readings of the records were made at 2-minute intervals from 04h 00m of May 23, 1960 (UT) up to 0.1 mm corresponding to about 0.25 μgal in gravity difference, obtaining values of 1480 in reading. In that case, since the speed of motion of the recording paper was 8 mm/hour, direct readings for the records were impossible. Therefore, they were made by enlarging the original record to be 3 centimetres the distance between hour-marks.

In order to remove the gravity variation due to earth tides and drift of the gravimeter, the high-pass filtering described by W. Munk and others (**33**) is applied to all the read values (**34**).

Now, let x_t and y_t be the read value and filtered result at the time of t_t respectively, y_t is expressed by the following formula,

$$y_t = x_t - \frac{1}{m^2}[x_{t-m+1} + 2x_{t-m+2} + 3x_{t-m+3} + \cdots\cdots + (m-1)x_{t-1} + mx_t + (m-1)x_{t+1} + \cdots\cdots + x_{t+m-1}]. \tag{5.1}$$

The filter (5.1) has an ability to exclude fully the waves with frequencies lower than f, giving the following formula,

$$2fm\Delta t = 2, \tag{5.2}$$

where Δt is the time interval between two readings of x_t. In the present case Δt being 2 in the minute. Since the waves with periods lower than 2 hours have to be removed, f is given in minutes as

$$f = \frac{1}{2\times 60}, \tag{5.3}$$

so that

$$m = 60 \tag{5.4}$$

In practical calculation of y_t, it was simpler to apply the following iterative scheme (5.5) derived from the formula (5.1) than to apply the formula (5.1) itself.

$$\left.\begin{aligned} v_{t+1} &= v_t - x_{t-m} + 2x_t - x_{t+m} \ , \\ y_{t+1} &= y_t + x_{t+1} - x_t + \frac{1}{m^2} v_{t+1} \end{aligned}\right\} \tag{5.5}$$

But y_{60}, the initial value of y, was calculated from the formula (5.1) and v_{61}, that of v, by the following formula

$$v_{61} = x_1 + x_2 + x_3 + \cdots\cdots + x_{60} - (x_{61} + x_{62} + x_{63} + \cdots\cdots + x_{120}) . \tag{5.6}$$

59 read values at each end of the time range disappeared in the filtering process and consequently total numbers of y_t amounted to

$$1480 - 59 \times 2 = 1362 . \tag{5.7}$$

Then, rewritting

$$y_j = y_{t-59} , \tag{5.8}$$

j being an integer greater than 1 and less than 1362. y_j thus obtained were used for a Fourier analysis successively made. Calculations in both filtering process and Fourier analysis were carried out by an electronic computing machine 'NEAC-2203' with the help of H. Takeuchi and M. Saito of Tokyo University.

Next, values of A_n, B_n, C_n^2 and ϕ_n were calculated by the following formulae (5.9) using y_j,

$$\left.\begin{aligned} A_n &= \sum_{j=1}^{J} y_j \cos 2\pi \frac{nj}{J} . \\ B_n &= \sum_{j=1}^{J} y_j \sin 2\pi \frac{nj}{J} , \\ C_n^2 &= A_n^2 + B_n^2 , \\ \phi_n &= \tan^{-1} \frac{B_n}{A_n} , \end{aligned}\right\} \tag{5.9}$$

where $J = 1362$ and $-\frac{\pi}{2} \leqq \phi_n \leqq \frac{\pi}{2}$ (ϕ_n in radians).

In practical calculation, the value of period T was also obtained by

$$T = \frac{J\Delta t}{n} = \frac{2J}{n} \tag{5.10}$$

For lack of memory of the electronic computing machine used in the present analysis, y_j were divided into two parts and their Fourier analysis was made for each part. For the latter half of y_j, A_n' and B_n' were calculated by the following formulae

$$\left.\begin{aligned} A_n' &= \sum_{j=j_0+1}^{J} y_j \cos\frac{2\pi n(j-j_0)}{J}, \\ B_n' &= \sum_{j=j_0+1}^{J} y_j \sin\frac{2\pi n(j-j_0)}{J} \end{aligned}\right\} \tag{5.11}$$

Multiplying these values by a certain coefficient and combining the results with A_n and B_n obtained from the former half of y_j, analytical results for all parts were obtained.

The formulae (5.11) gave Fourier coefficients at any fixed time j_0+1 being its starting-point. As the causes for partition of the data, one was due to the lack of memory of the electronic computing machine as mentioned above, and the other so as to obtain a value of dissipation constant Q (**27**, **30**, **35**) for any fixed frequency.

4. Results of spectral analysis

The first value of y_j obtained through the filtering process corresponded to 05h 58m of May 23, 1960 (UT) and their total numbers amounted to 1362. By using these values y_j, T in (5.10) and A_n, B_n and C_n^2 in (5.9), were calculated. The obtained results were shown in Table 5.3. Calculation was carried out from $n=41$ to $n=80$. In Table 5.3, one unit of A_n, B_n and C_n corresponded to 2 mm on the registrogram and also to about 5.1 μgal in gravity difference. Relation between power C_n^2 and wave number n, was shown in Fig. 5.2. The spectral analysis was also made for every six of n from $n=41$ to $n=155$. The relation between C_n^2 and n, for a range of $n=71$ to $n=155$, was shown in Fig. 5.3. Since there remained considerable influence of earth tides at a range where n being smaller than 40, and the error was large at a range where it exceeded 150, no significant result was presumably able to be obtained even if an analysis was made. Therefore, no calculation was made for those ranges.

Owing to rough time interval between two readings and lack of total numbers of the readings, real and visionary peaks were mingled in the spectral peaks shown in Figs. 5.2 and 5.3. Tukey's filter (**36**) was then applied to the power spectra obtained above so as to distinguish increasingly the real peak and to eliminate the visionary one. It was a kind of moving means

Table 5.3. Values of T, A_n, B_n and C_n^2

n	T (minute)	A_n	B_n	C_n^2
41	66.44	− 87.01	17.52	7878
42	64.86	73.82	75.83	11200
43	63.35	− 70.84	87.60	12690
44	61.91	5.178	39.74	1606
45	60.53	76.83	57.23	9178
46	59.22	− 12.01	100.4	10220
47	57.96	−113.2	− 9.998	12920
48	56.75	− 46.28	− 11.28	2269
49	55.59	− 4.861	− 30.66	963.7
50	54.48	53.95	53.61	5785
51	53.41	− 54.18	56.27	6101
52	52.38	50.93	7.670	2653
53	51.40	12.66	− 24.06	739.1
54	50.44	− 19.17	8.434	438.8
55	49.53	12.32	7.481	207.8
56	48.64	12.00	30.67	1085
57	47.79	− 40.13	111.2	13980
58	46.97	13.45	− 48.68	2550
59	46.17	71.89	− 18.82	5523
60	45.40	− 27.73	− 17.51	1075
61	44.66	− 20.46	20.51	839.3
62	43.94	− 4.220	50.40	2558
63	43.24	− 39.34	81.45	8181
64	42.56	18.70	52.76	3134
65	41.91	30.03	56.66	4112
66	41.27	− 60.57	31.43	4657
67	40.66	49.71	13.69	2659
68	40.06	− 23.29	4.760	565.2
69	39.48	− 62.67	−118.6	17990
70	38.91	8.442	− 21.50	533.4
71	38.37	48.16	− 35.18	3557
72	37.83	33.25	1.192	1107
73	37.32	43.81	61.63	5717
74	36.81	− 27.17	20.56	1161
75	36.32	− 59.32	− 17.29	3818
76	35.84	65.80	− 27.92	5109
77	35.38	− 60.41	− 10.52	3760
78	34.92	18.81	− 5.751	386.9
79	34.48	45.67	11.68	2222
80	34.05	34.25	32.64	2239

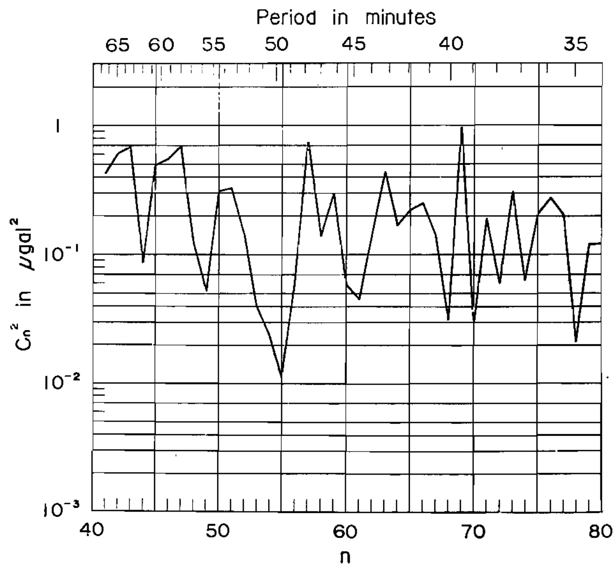

Fig. 5.2. Power C_n^2 for periods of 65–35 minutes.

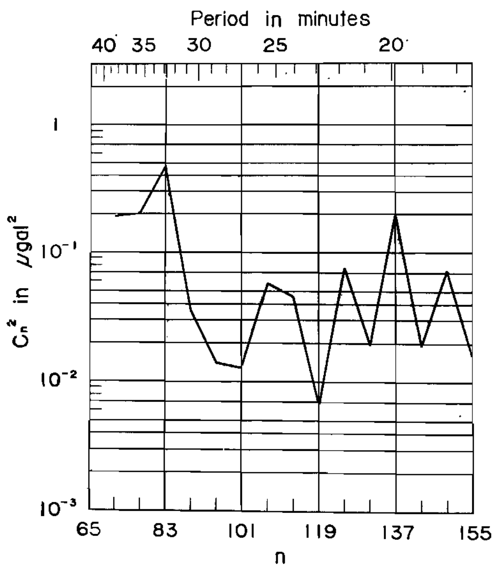

Fig. 5.3. Power C_n^2 for periods of 35–18 minutes.

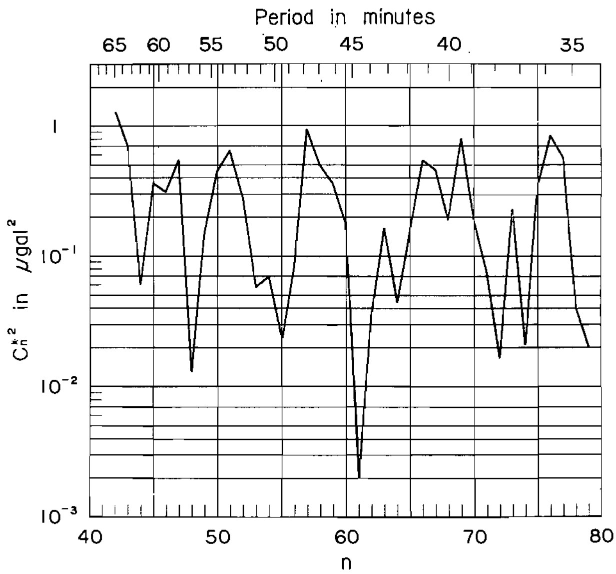

Fig. 5.4. Smoothed power $C_n{}^{*2}$ for periods of 65–35 minutes.

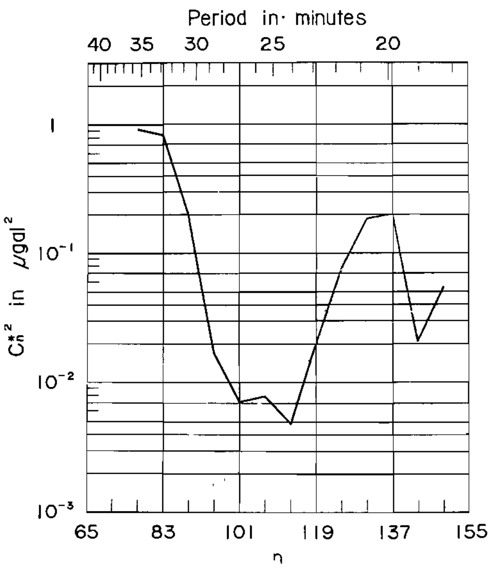

Fig. 5.5. Smoothed power $C_n{}^{*2}$ for periods of 35–18 minutes.

and its process was expressed by the following formulae

$$\left.\begin{aligned} A_n^* &= \frac{1}{4}(-A_{n-1}+2A_n-A_{n+1})\,. \\ B_n^* &= \frac{1}{4}(-B_{n-1}+2B_n-B_{n+1})\,, \\ C_n^{*2} &= A_n^{*2}+B_n^{*2} \end{aligned}\right\} \tag{5.12}$$

Smoothed spectra C_n^{*2} corresponding to Figs. 5.2 and 5.3 thus obtained were shown in Figs. 5.4 and 5.5 respectively. In case of making out the Fig. 5.5, the suffix $n\pm1$ was replaced by $n\pm6$ on the right-hand side of the expressions A_n^* and B_n^* in (5.12).

Taking accuracy of readings from the original registrogram into consideration, the following periods could be picked up from these figures as power spectral peaks in the present analysis:

58.0, 53.4, 47.8, 43.2, 41.3, 39.5, 37.3, 35.8, 25.5, 20.8 and 19.9 minutes.

Table 5.4. Observed and theoretical periods of the earth's free oscillations

Mode	Theoretical	Observed	Period obtained in foreign countries				
			(a)	(b)	(c)	(d)	(e)
${}_0S_2$	53.5	53.4	55.0 , 52.8	54.7 , 53.1	53.4	54.4	
		47.8		46.2	46.7		
${}_0T_2$	43.4	43.2		42.3	44.8		
		41.3			41.3		
		39.5					
		37.3			36.7		
${}_0S_3$	35.3	35.8	35.9 , 35.2	35.9 , 35.2	35.8	35.4	
${}_0T_3$	28.1			28.6	28.5	28.8	
${}_0S_4$	25.5	25.5	25.9	25.8	25.8	25.5	25.8
${}_1S_2$	24.7		24.7	24.5	24.8		24.0
${}_0T_4$	21.5			21.8	21.9	21.5	21.6
${}_0S_0$	20.7	20.8	20.5			20.4	
${}_0S_5$	19.8	19.9	19.8	19.8	20.0	19.6	

(unit in minutes)

(a) N. F. Ness, J. C. Harrison & L. B. Slichter (30)
(b) H. Benioff, F. Press & S. Smith (27)
(c) L. E. Alsop, G. H. Sutton & M. Ewing (28)
(d) B. P. Bogert (29)
(e) W. Buchheim & S. W. Smith (31)

5. Discussion

(1) Periods of the earth's free oscillations

Peaks of the power spectra obtained by the present spectral analysis are shown in Table 5.4. In this table, periods of the earth's free oscillations observed in America and Germany at the time of the same Chilean earthquake and those predicted by theoretical investigations, are also collectively shown. In Table 5.4, there is no description for spectral peaks with a period longer than 54 minutes, but there are two spectral peaks at that period as shown in Fig. 5.4. They are presumably appeared due to a fact that one time of operation of the high-pass filter, used to eliminate the earth tides and the instrumental drift, is not enough to remove those two peaks.

Spheroidal oscillations can only essentially be detected by a gravimetric observation. In fact, according to the results obtained with a LaCoste-Romberg tidal gravimeter by N. F. Ness and others (**30**), only the spheroidal mode has been detected. But, according to the observational results obtained by the author, periods corresponding to torsional modes T_3 and T_4 have not been observed, while a period corresponding to a torsional mode T_2 has clearly been observed. The magnitude of power corresponding to T_2 is the smallest in a range of $n=40$ to $n=80$. It is necessary to investigate in detail whether this peak, corresponding to T_2, is real or a visionary one.

Among the spectral peaks obtained above, there exist several unassigned theoretically with periods of 47.8, 41.3, 39.5 and 37.3 minutes. Among them, the magnitude of power at 37.3-minute period is small, but that of the others is not always small in comparison with that of peaks corresponding to spheroidal modes. In particular, a peak at 47.8-minute period is the largest within a range of period spectrally analysed by the author. This peak may presumably appear under an influence of operation of a thermostatically controlling apparatus in the observation room, because of its period being about 48 minutes. As shown in Table 5.4, H. Benioff and others (**27**) have detected a peak with 46.2-minute period from strain seismograms obtained at two stations, Isabella and Ñaña, and L. E. Alsop and others (**28**) also detected that of 46.7-minute period from seismograms obtained at Palisades. It is a problem to be solved in the future whether there exists actually an oscillation mode with a period of 46 to 48 minutes or it is an apparent one caused by changes of the meteorological and other disturbing elements.

There then exists a spectral peak at 20.8-minute period in the present analysis, but it is difficult to determine which of S_0 and T_4 corresponds to it, because time interval between two readings is 2 minutes. Judging from accuracy of the Askania gravimeter, a radial mode S_0 must be observed sufficiently by using the gravimeter as an altimeter, and it is therefore suspected that the peak at that period corresponds to S_0. In fact, the earth's free oscillation of S_0 type is clearly recognized in results obtained with a LaCoste-Romberg tidal gravimeter by N. F. Ness and others (**30**).

Although there leaves some room for further consideration in detail, the periods of peaks corresponding to S_0, S_2, S_3, S_4 and S_6 obtained by the present spectral analysis, are in good agreement with those obtained in America at the time of the same earthquake and are also in splendid agreement with those predicted by the theoretical investigations. This means that knowledges concerning the internal constitutions of the earth obtained through a propagation of seismic waves and phenomenon of the earth tides, are exceedingly correct.

In the following, some detailed discussions concerning the fundamental spheroidal mode S_2 of the earth's free oscillations with a period of 53.4-minutes, are made.

(2) Magnitude of the power for 53.4-minute period

From Table 5.3, a magnitude of power at 53.4-minute period is

$$C_n^2 = 6.1 \times 10^3 \tag{5.13}$$

Now, putting

$$y_j = \sum_{n=0}^{J/2} \left(a_n \cos 2\pi \frac{nj}{J} + b_n \sin 2\pi \frac{nj}{J} \right), \tag{5.14}$$

where a_n and b_n are Fourier coefficients to be determined, they are generally expressed as follows:

$$\left. \begin{aligned} a_n &= \frac{2}{J} \sum_{j=1}^{J} y_j \cos 2\pi \frac{nj}{J}, \\ b_n &= \frac{2}{J} \sum_{j=1}^{J} y_j \sin 2\pi \frac{nj}{J}, \\ c_n^2 &= a_n^2 + b_n^2 . \end{aligned} \right\} \tag{5.15}$$

Combining (5.15) with (5.9), the following relations are obtained.

$$\left.\begin{aligned} a_n &= \frac{2}{J} A_n \,, \\ b_n &= \frac{2}{J} B_n \,, \\ c_n{}^2 &= \left(\frac{2}{J}\right)^2 \cdot C_n{}^2 \,. \end{aligned}\right\} \tag{5.16}$$

Since one millimetre on the recording paper corresponds to 2.5489 μgal in gravity change (**32**), one unit in values of the reading corresponds to

$$2.5489\ \mu\text{gal/mm} \times 2\ \text{mm} = 5.0978\ \mu\text{gal}\,. \tag{5.17}$$

Therefore, the magnitude of power corresponding to (5.13) is calculated in microgals as follows:

$$\begin{aligned} Power &= \left(\frac{2}{J} \times 5.0978\right)^2 \cdot C_n{}^2 \\ &= 0.34\ \mu\text{gal}^2\,. \end{aligned} \tag{5.18}$$

On the other hand, according to the results obtained by N. F. Ness and others (**30**), an energy density for the same oscillation with 54-minute period, is about 0.5 μgal²/cph. Multiplying this value by 1.1 cph, which is a frequency corresponding to 54 minutes, the corresponding power after their results, is calculated as follows:

$$Power_{\text{Ness and others}} = 0.55\ \mu\text{gal}^2\,. \tag{5.19}$$

The value in (5.19) is one to be compared with that in (5.18). Except a special case of which either of the two stations is situated on a nodal line of the oscillation, both values must in principle be of the same order of magnitude. From both values (5.18) and (5.19), it is suspected that the spectral peak with 53.4-minute period obtained above, corresponds to the fundamental spheroidal mode of the earth's free oscillations.

(3) Generation mechanism of the earthquake

As already described in section 3, the initial time of readings from the records is 04h 00m of May 23. 1960 (UT). Since 59 read values at the beginning of time range disappear in the high-pass filtering process, the time of origin of the spectral analysis is 05h 56m of May 23. As shown in Table 5.3, coefficients of cosine and sine components of the gravity variation with 53.4-minute period, are negative and positive respectively and their absolute values are almost equal. This means that a time of zero gravity variation is about $\pi/4$ (that is. 6.7 minutes) behind the origin time 05h 56m of May

23 in the present spectral analysis. A sign of the gravity variation changes from negative (decrease) to positive (increase) at the time of zero gravity variation 06h 02.7m of May 23 thus determined. Searching the time of a similar phase near the origin time of the Chilean earthquake, 19h 11m of May 22 determined by seismometric observations, it is 19h 21.9m of May 22 going back 12 periods ($53.4\times 12=640.8$ minutes) from 06h 02.7m of May 23 obtained above. The time of origin of this earthquake is 10.9 minutes preceding that time. At the origin time 19h 11m of May 22, the gravity variation with 53.4-minute period, is negative (decrease) and its absolute amplitude is nearly maximum (-0.58 μgal).

Taking into consideration the excitation of the earth's free oscillations, it seems natural that an amplitude of the gravity variation has its extreme at the origin time of the earthquake. The mode of 53.4-minute oscillation is of S_2^1 type as in the fault plane problem in seismology. Similar studies of the earth's free oscillations as in the present article, will throw light on study of earthquake generation mechanisms.

(4) Amplitude of vertical displacement

The next problem is to determine an amplitude of vertical displacement corresponding to the gravity variation of the 53.4-minute period.

Extracting a square root of (5.18), an absolute value of the gravity variation with the 53.4-minute period is

$$\Delta g=0.58\ \mu\text{gal}=0.58\times 10^{-6}\ \text{gal}\,. \tag{5.20}$$

In the present case, the gravimeter is always moved with the earth's surface. Then, assuming ρ(density at near surface of the earth) $=2.67$ gr/cm^3 and ρ_m (mean density of the earth) $=5.53$ gr/cm^3, a gravity variation Δg originated from vertical displacement of the observation station, is approximately given by

$$\begin{aligned}\Delta g&=(3.086-1.118)\times 10^{-6}\ r\ \text{gal}\\&=1.968\times 10^{-6}\ r\ \text{gal}\,,\end{aligned} \tag{5.21}$$

where r is the vertical displacement of the observation station measured in centimetres. Assuming that the gravimeter works as an altimeter, the amplitude of vertical displacement can simply be calculated by putting (5.20) equals to (5.21). Its result is

$$r = 0.29 \text{ cm} . \tag{5.22}$$

The equation (5.21) is a relation to be acceptable when the vertical motion is quasi-statical.

Next, let one consider frequency characteristics of the gravimeter. Since the exact values of electric resistance of photocell and damping constant of the instrument are unknown, it is difficult to know details concerning the frequency characteristics of the gravimeter. But, the following data are known to the first order by experience.

a. The period T_1 of the gravimeter's pendulum lies between 12 and 15 seconds.

b. The period T_2 of galvanometer for recording is 20 seconds.

c. The damping constant h_1 of the gravimeter is larger than 1.0 but not extraordinarily large.

d. The damping constant h_2 of the galvanometer is considerably large.

Then the following period T and damping constant h for the gravimeter and galvanometer, are respectively adopted :

$$\left.\begin{array}{lll} \text{Gravimeter} & : T_1 = 12 \sim 15 \text{ sec} , & h_1 = 1.0 \sim 10 \ ; \\ \text{Galvanometer} & : T_2 = 20 \text{ sec} \quad , & h_2 = 10 \sim 120 \ . \end{array}\right\} \tag{5.23}$$

Relation between the relative magnification α of the gravimeter and period of gravity variation is shown in Fig. 5.6, while that between magnification β of the gravimeter as a seismometer and period of gravity variation, is also shown in Fig. 5.7.

In Fig. 5.6, the response curve in case of $T_1 = 12$, is in almost perfect agreement with that of $T_1 = 15$, consequently only the former is shown in the figure. As can easily be seen from Figs. 5.6 and 5.7, there are considerable

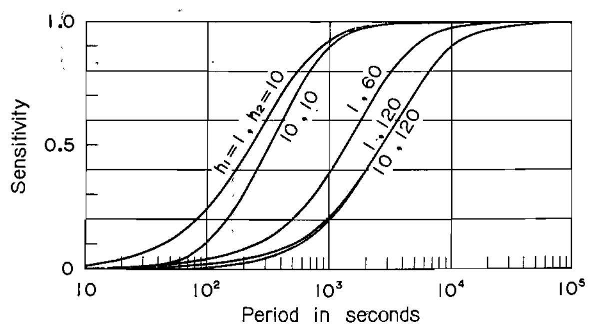

Fig. 5.6. Calculated response curve of the Askania gravimeter No. 111.

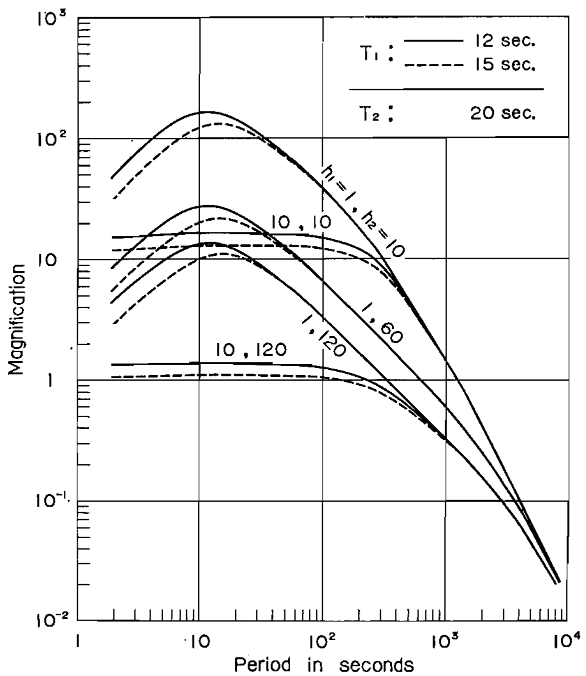

Fig. 5.7. Calculated magnification curve of the Askania gravimeter No. 111 as a seismometer.

differences between the cases of $h_2=10$ and $h_2=120$, while there is no difference between the cases of $h_1=1$ and $h_1=10$ for oscillations with a long period. Furthermore, there is no difference between the cases of $T_1=12$ and $T_1=15$ for those oscillations. Then the following case is also shown in Figs. 5.6 and 5.7 as a mean response curve, and its curve is used in the succeeding consideration.

$$\left.\begin{array}{ll}\text{Gravimeter} & : \; T_1=12\text{ sec}, \quad h_1=1.0\,; \\ \text{Galvanometer} & : \; T_2=20\text{ sec}, \quad h_2=60\end{array}\right\} \tag{5.24}$$

From Figs. 5.6 and 5.7, values of α and β for an oscillation of 53.4-minute period are obtained as follows :

$$\left.\begin{array}{l}\alpha=0.80\,, \\ \beta=0.12\,.\end{array}\right\} \tag{5.25}$$

When the values given in (5.25) are used in calculation, an error in their estimation is smaller than 20%.

Now, when the vertical motion is periodical, the equation (5.21) must be replaced by

$$\Delta g = 1.968 \times 10^{-6}\ \alpha\ r\ \text{gal}\,, \tag{5.26}$$

and the gravity variation due to an action of the gravimeter as a seismometer is expressed for an oscillation with a period T by

$$\Delta g = \beta\left(\frac{2\pi}{T}\right)^2 r \tag{5.27}$$

Since the gravity variations (5.26) and (5.27) work in an opposite sense, one has

$$\Delta g = 1.968 \times 10^{-6}\ \alpha\ r - \beta\left(\frac{2\pi}{T}\right)^2 r \tag{5.28}$$

Inserting the values of (5.20), (5.25) and $T = 53.4 \times 60$ into (5.28), an amplitude of the vertical motion with 53.4-minute period is calculated as follows:

$$r = 0.52\ \text{cm} \tag{5.29}$$

6. Summary

Oscillations excited by the great Chilean earthquake of May 22, 1960 were simultaneously recorded during a few days with two Askania gravimeters at Kyoto where they had been at work for the purpose of observing the tidal variation of gravity. In the present article, a trial to detect free oscillations of the earth from records obtained with the Askania gravimeter No. 111, was in detail described. Readings of the records were made at 2-minute intervals from 04h 00m of May 23 (UT) up to about 0.25 μgal and values of 1480 readings were obtained. These read values thus obtained were filtrated by a high-pass filter and analysed by Fourier's method by using an electronic computing machine 'NEAC-2203'. After various considerations on the basis of results obtained by the spectral analysis, the following conclusions were obtained.

(1) The earth's free oscillations of spheroidal modes S_0, S_2, S_3, S_4 and S_5 were clearly detected within ranges of period between 18 and 60 minutes. These observed periods were in good agreement with those obtained in America and also with periods predicted by theoretical investigations.

(2) The spectral peak at a period of 53.4 minutes was one correspond-

ing to the fundamental spheroidal oscillation of the earth, and its magnitude was about 0.34 μgal^2 in power. This value was of the same order of magnitude as that obtained by N. F. Ness and others at Los Angeles.

(3) The gravity variation, due to the earth's free oscillation of 53.4-minute period, was about 0.58 μgal in amplitude and attained its negative maximum at the origin time of the Chilean earthquake, that is, 19h 11m of May 22, 1960 (UT).

(4) The amplitude of vertical displacement corresponding to the gravity variation of 53.4-minute period was calculated as follows:

When the motion was quasi-statical : 0.29 cm

When the motion was periodical : 0.52 cm .

The results obtained by the present research were not so splendid as those obtained in America at the time of the same Chilean earthquake. In the present spectral analysis, some peaks theoretically unassigned were also detected. It was presumably caused by a deficiency of number of read values. Since the speed of motion of the recording paper was 8 mm/hour, it was possible to read at every minute, by enlarging the original records, as in America. It was important to compare in detail the power spectrum, obtained by the same method as in the present analysis based on a recorded curve at quiet periods, with that obtained above. Since a vibration excited by the earthquake was continuously recorded during a period of several days at least, it was possible to investigate a damping mode of free oscillations of the earth and value of the dissipation constant Q, by extending a range of readings and by analysing its range dividing into some parts. Furthermore, it was necessary to analyse records obtained with the Askania gravimeter No. 105 by the same method as for the present ones, because only the records obtained with the Askania gravimeter No. 111, were analysed in the present case.

More data were available and their reading at every minute was possible, but a hire of the electronic computing machine to treat the data was limited. Under these circumstances, a further detailed discussion could not be made. A detailed investigation concerning these various problems would be made, when the circumstances permitted.

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